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3D Inversion of Gravity Data for Obama Geothermal Field.

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ABSTRACT

In this paper we have utilize fast 3D gravity inversion(a certain iterative procedure known as Lanczos bidiagonalization)technique that is able rapidly recovering 3D density distributions extracted from measured gravity anomalies. , the new method was applied to real gravity data obtained from the Obama geothermal field .Our 3D model shows the shape of the anomaly source in the study area. The deduced model of substratum agrees well with previous geological studies.

Keywords: inversion, gravity data, Obama geothermal field

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INTRODUCTION

Inversion is in fact described right here is considered an automated numeral processes utilized build a model of about subsurface physical property(density) variations by using measured data and all of the prior important information independent of about measured data. Quantitative interpretation will be performed by drawing geologic interpretations from the original and inverted models. A model is in fact which is parameterized to explain the source geometry or is described using a distribution of a new physical property, for instance density or magnetic susceptibility contrast. [1].

The main goal of this paper is to utilize fast 3D gravity inversion technique that is able rapidly recovering 3D density distributions extracted from measured gravity anomalies. To achieve this, the survey area is divided right into a large number of rectangular prisms within a network which have uncertain densities. To realize that goal, a certain iterative procedure known as Lanczos bidiagonalization is utilized.

The Lanczos bidiagonalization regularization algorithm appeared to be suggested by [2-3] to use on the simplistic computation that is approximate solutions of enormous linear systems of equations along with ill-posed forward operator matrix. The best rule can be referred to in [5]. This formula needs k steps by applying the Lanczos bidiagonalization of matrix A [1-4-5-6]. A comprehensive structure of a given algorithm summarized inside [6] and [5] in detail. The original system of equations is substituted with a system of lowered dimension to improve the speed of a given solution procedure greatly while being able to solve the original problem with a high degree of accuracy [4]. A least-squares QR (LSQR) method is applied to select the best value of a regularization parameter. The Lanczos bidiagonalization was performed using the MATLAB-based regularization tools, which is open source [7].finally, the new method was applied to real gravity data obtained from the Obama geothermal field.

LANCZOS BIDIAGONALIZATION OF A REGULAR MATRIX

For a rectangular matrix A of size m by n, a sequence of vectors $u_j \in R^m$ and $v_j \in R^n$ and scalars α_j and β_j for $j=1,2,\dots,k$ can be calculated through the following process after k iterations of Lanczos bidiagonalization [2]:

1-Given a starting vector $p_0 \in R^m$, set $\beta_1 = \|p_0\|$, $u_1 = p_0/\beta_1$, and $v_0=0$

2. For $j > 1$ to K

$$R_j = A^T U^j - \beta_j v_{j-1}$$

$$\alpha_j = \|r_j\|_2$$

$$v_j = r_j/\alpha_j$$

$$P_j = A v_j - \alpha_j u_j \quad j=1,2,\dots,k$$

$$\beta_{j+1} = \|P_j\|_2$$

$$u_{j+1} = P_j/\beta_{j+1}$$

Here, vectors u_j and v_j resulting from the above Lanczos bidiagonalization process are known as the Lanczos vectors and they satisfy the recurrence relations.

$$\alpha_j v_j = A^T u_j - \beta_j v_{j-1}$$

$$\beta_{j+1} u_{j+1} = A v_j - \alpha_j u_j$$

Which can be represented in compact matrix form as

$$A V_k = U_{k+1} B_k$$

$$A^T U_{k+1} = V_k B_k^T + \alpha_{k+1} v_{k+1} e_{k+1}^T$$

Where $U_{k+1}=(u_1, u_2, \dots, u_{k+1})$ $V_k=(v_1, v_2, \dots, v_k)$ e_i is the i^{th} Column of a unit matrix with the appropriate dimension, and B_k is a lower bidiagonal matrix.

$$B_k = \begin{Bmatrix} \alpha_1 & & & & \\ \beta_2 & \alpha_2 & & & \\ & \beta_3 & & & \\ & & \alpha_k & & \\ & & & \beta_{k+1} & \end{Bmatrix}$$

In the case of exact arithmetic, the Lanczos vectors are orthonormal so that

$$U_{k+1}^T U_{k+1} = I_{k+1}, \text{ and } V_k^T V_k = I_k$$

Where I_k is the $K \times k$ identity matrix.

RESOLUTION MATRIX ESTIMATION BASED ON LSQR

For the matrix equation $Ax = b$, the LSQR algorithm of Paige & Saunders (1982) is based on the Lanczos bidiagonalization process described above with the starting vector p_0 equal to the data vector b . In this case, $U_{k+1} = b$. A solution x_k is sought in k dimensional Krylov space spanned by V_k to minimize $\|Ax - b\|$. The solution can be written in the form $x_k = V_k y_k$ and it then follows that.

$$\min_{x \in R^k} \|Ax - b\| = \min_{y \in R^k} \|AV_k y - b\| = \min_{y \in R^k} \|U_{k+1}(B_k y - \beta_1 e_1)\|$$

Using the orthogonality of U_{k+1} , the minimization of $\|Ax - b\|_2$ is satisfied by choosing y_k to be solution to the least squares problem of $\min \|(B_k y - \beta_1 e_1)\|_2$. Therefore, the LSQR algorithm is a conjugate gradient method where a good solution in the Krylov subspace spanned by Lanczos vectors can quickly be obtained with a small number of iterations because each Lanczos vector points to a steep descent direction in subspace of eigenvectors [8]

Suppose the SVD of B_k is $B_k = P_k D_k Q_k^T$. then we can represent A from eq (12) as

$$A = U_{k+1} B_k V_k^T = (U_{k+1} P_k) D_k (Q_k^T V_k^T).$$

Where non-zero diagonal values of D_k are called the Ritz values of matrix A , $U_k = U_{k+1} P_k$ and $V_k = V_k Q_k$ are Ritz vectors. If the Ritz values and corresponding Ritz vectors are close to the true singular values and singular vectors of matrix A eq (16) give the truncated SVD of matrix A . Since there exists a very efficient algorithm to calculate the SVD of the lower bidiagonal matrix B_k , we could efficiently calculate the SVD of A by means of the Lanczos bidiagonalization process. This is the basis for the LAQR- based resolution matrix estimation proposed by [8] and [10] as well as the approximate SVD estimation using the PROPACK package .[11]

Gravity data

The Obama geothermal field was covered by gravity surveys as a routine method for monitoring and evaluating the geothermal reservoir. A density of 2.3 g/cm^3 was used to produce the Bouguer anomaly map of the study area (Figure 1). Visual inspection of Figure (1) shows that the area is characterized by positive gravity values covering the whole area, ranging between 11.2 and 13.5 mgal, and increasing in the eastern and southern part of the map area. This could be related to the low gradient in the subsurface structure. The observed variations in the anomalies reflect the half graben structure associated with the volcano-tectonic depression zone of Shimabara Peninsula. [12]

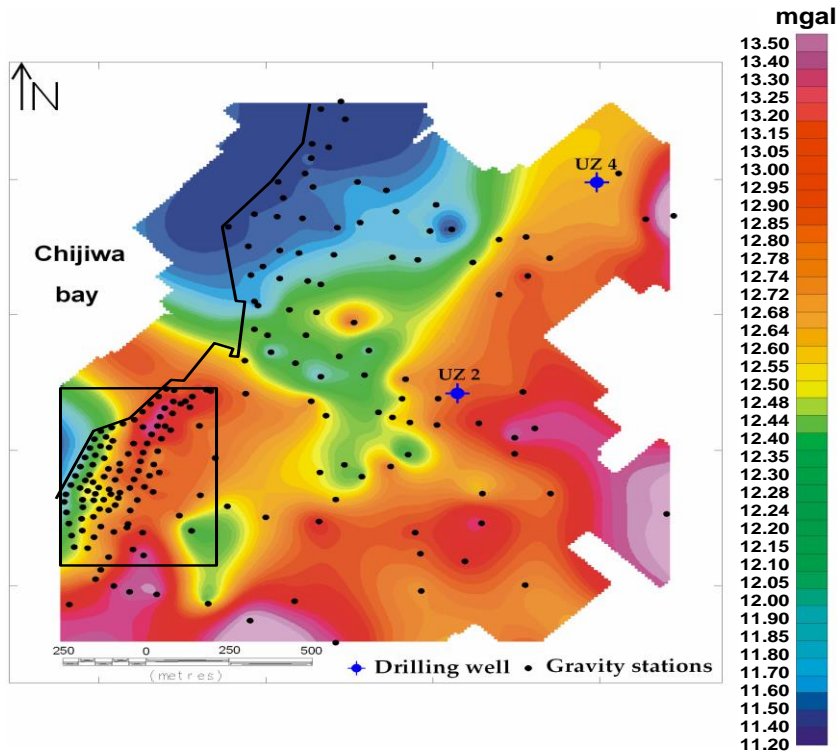


Figure (1): Bouguer anomaly map of the Obama field, $\rho = 2.3 \text{ g/cm}^3$. The black rectangle indicates the southern part of the study area. The black line indicates the coastline. [12]

Inversion method

The purpose of this study is to know the structure of substratum beneath Obama area for a good comprehension of hydro-geothermal fluid dynamism within Obama. For this, inverse interpretation method of gravity anomalies is used [1] to approximate the structure of substratum. Density contrast between substratum and sediment rock is -0.2 g/cm^3 determined, basement rock with average density of 2.4 g/cm^3 composed by Pliocene (Neogene) formations and the sediment rock with average density of 2.2 g/cm^3 composed essentially by Quaternary formations. [12]. One of the main objectives of gravity study is interpretation of Bouguer anomaly data. The results are shown in Figure (2).

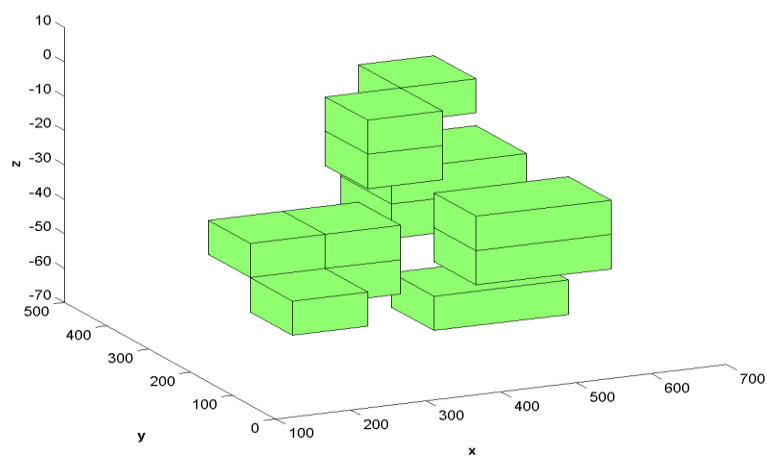


Figure (2): Three-dimensional model of basement rock structure beneath Obama associated with estimated hydro-geothermal flows and geological explanations.

The deduced model of substratum agrees well with previous geological studies. In Figure (2), wells are plotted on the 3D model.

CONCLUSION

A new method for the 3D inversion of gravity data that uses a Lanczos bidiagonalization method was used for our study area. The method used to decrease the instability and to guarantee the uniqueness of the solution is to combine geological and geophysical constraints (density values) into the inversion modeling. Field data tests show that our method is able to recover an anomaly source with different density contrasts. Moreover, our method is able to handle irregularly sampled data. Our 3D model shows the shape of the anomaly source in the study area. The deduced model of substratum agrees well with previous geological studies.

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